From climate risk to financial risk

The real option theory as a tool allowing to translate knowledge from climate science and climate economics into financial risk.

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1) Introduction: the concept of real options

2) Climate change and real options: the macro level

3) Climate change and real options: the micro level

1) Introduction: the concept of real options

- The Real Option theory allows to go beyond the static NPV approach
- The Real Option approach is a dynamic decision making tool which in principle allows for optimal investment choices
- The underlying asset of the Real Option is not a financial one, as it is the case with financial options, but a real investment.
- Therefore, usually the physical probability is used instead of the risk-neutral one.

- The investment is an opportunity and therefore perceived as an option.
- In a dynamic setting, real option models allow the following questions to be answered: when, how much and possibly where.

When should the investment optimally be undertaken?
How much should be invested or what is the value of the investment opportunity? and possibly, where should the investment take place?

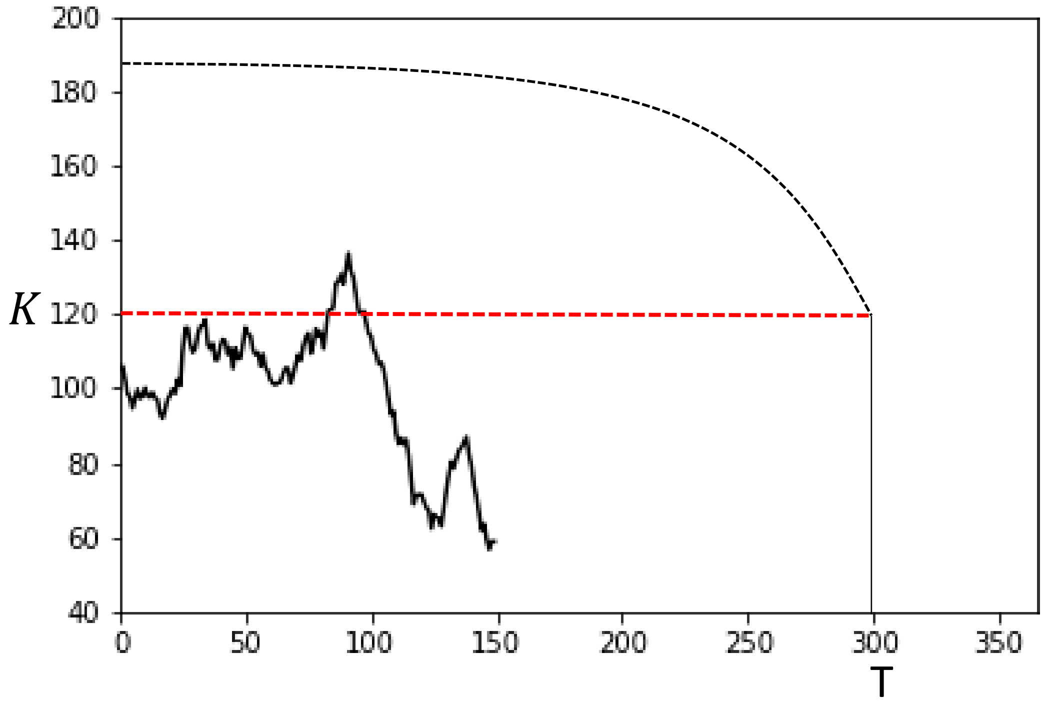
- These questions are obviously not independent.
  In case of inaccuracies regarding the time dimension, the value of the investment opportunity is likely to be misestimated.

- Working in a dynamic setting allows risk to be a key factor in Real Option models.
  This factor strongly impacts the answers to the three questions: when, how much and where.
  In standard Real Option models, an increase in the risk level might delay the investment decision and might induce a higher value of the investment opportunity.
  In the case of investments in global warming mitigation projects, this standard result is challenged.

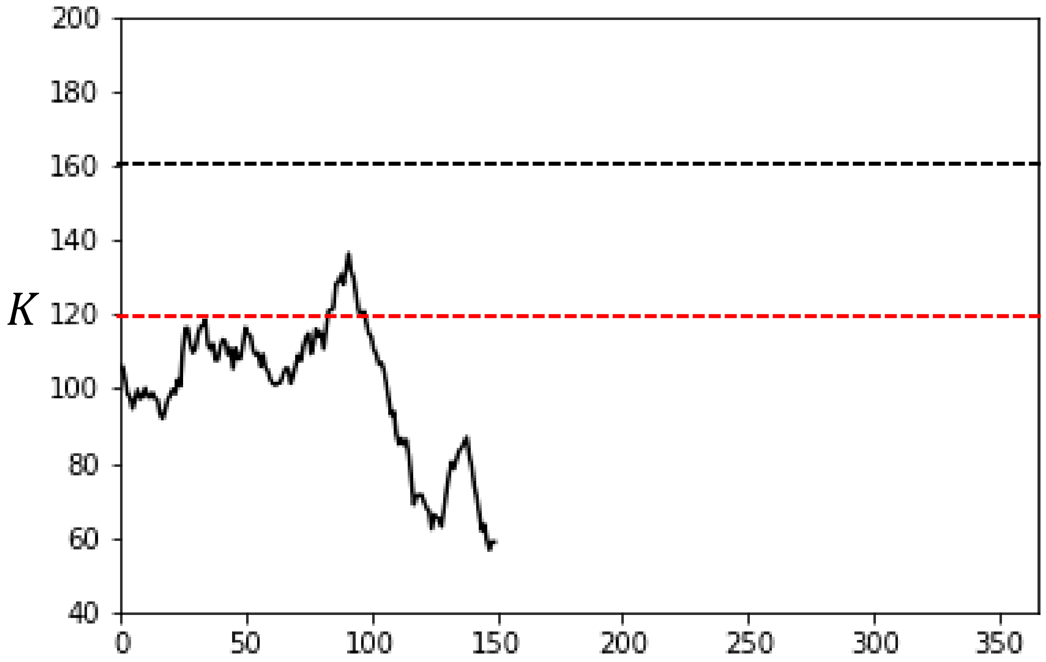
- The investment boundary is a concept which allows in particular an interplay between the time dimension and the risk level.
  At each point in time, it is the smallest expected sum of discounted cash-flows such that the investment opportunity should be undertaken.
  It depends on time and on the risk level.

The exercize boundary

The finite-life case



The perpetual case



- Extensions include the introduction of competition in this field.

Contrary to financial options, not only one, but various agents can own the same option to invest, in our setting to invest in a given technology in order to reduce CO_2 emissions and therefore mitigate global warming.

- There is a long list of books and papers in the field of real options.

The seminal book of Avinash K. Dixit and Robert S. Pindyck:
"Investment Under Uncertainty" Princeton U. press, 1994.
and their articles.

- “The orthodox theory of investment has not recognized the important qualitative and quantitative implications of the interaction between irreversibility, uncertainty, and the choice of timing. We will argue that this neglect explains some of the failures of that theory. “

- Risk versus uncertainty

Two examples of Real Option models in the context of climate change

- At the macro level, with the concept of risk

- At the micro one, where uncertainty is introduced

2) Climate change and real options: the macro level

Mitigating Global Warming: A Real Options Approach

Marc Chesney, Pierre Lasserre, Bruno Troja

Annals of Operations Research, 2017

Introduction

- This paper analyses optimal investment choices in mitigation and their timing, and seeks to find the temperature level at which a mitigation decision should be taken.
- It is assumed that a known mean global temperature level exists above which life might not be possible, i.e. an *environmental catastrophe* might happen.
- This *environmental catastrophe*, however, is generated only when the temperature process stays above a given threshold, or *tipping point* (Lenton et al., 2008), for some years.

Introduction

Mitigation and Adaptation, two different solutions to the same problem: Global Warming.

- Mitigation aims to reduce CO_2 emissions in the atmosphere, thus preventing the effects of global warming and climate change, *ex-ante*.
- Adaptation aims to reduce the impacts of climate change, *ex-post*.

Literature Review

- A substantial part of the economic climate change literature consists of integrated assessment models (IAM).
- Economic models on climate change and their outcomes have been investigated by Nordhaus and Boyer (2003) and Tol (2002). Nordhaus and Boyer (2003) developed a model called RICE, Regional Integrated model of Climate and Economy, which is an improvement on the famous DICE model (Dynamic Integrated model of Climate and Economy; Nordhaus (1992)).

Literature Review

- One of the most important studies on the effects of climate change on world GDP is that of Stern (2007). This author calculates the monetary impacts of inaction (or insufficient action) on the global economy. Stern (2007) found that due to inaction, the world may lose up to 5% or even more of its aggregate GDP each year. These costs are very high compared to what the author calculated as the amount needed to combat climate change, i.e., about 1% of world GDP if carbon concentration is stabilized at around 550 ppm.

Literature Review

- Pindyck (2015) claims that "IAMs, however, simply cannot account for catastrophic outcomes".
- Prieur et al. (2011) and Amigues and Moreaux (2013) introduced a threshold catastrophic temperature as the key element of an economic climate change model where the catastrophe causes infinitely large damage. While they used a dynamic but non-stochastic framework, Tsur and Zemel (2008) also consider the possibility of a catastrophic climate event in a stochastic environment.
  - Baranzini et al. (2003)
  - Lemoine and Traeger (2014)
  - Naevdal (2006)
  - Naevdal and Oppenheimer (2007)
  - Keller et al. (2004)

Possible Catastrophes

- In this paper, we model a climate catastrophe as an irreversible event of such magnitude and with such manifestations that it amounts to the end of society as we knew it before the catastrophe.
- Although this is by design an extreme representation of a climate catastrophe, it is not without scientific basis.
  - Dakos et al. (2008) and Lenton et al. (2008) find that a deviation from a threshold temperature sustained over time is capable of inducing dramatic changes to the environment.
  - Lenton et al. (2008) identified several policy-relevant tipping elements, i.e., events or climate states that could keep the temperature process above a certain threshold over a long period of time.

Real Options

- The environmental real options approach is based on the premise that, and applies when, environmental policies involve committing resources for the long term and are irreversible due to institutional and other constraints. Under such conditions, environmental policies are best modeled as once-and-for-all (or long-term) decisions (Pindyck (2000); Insley (2002); Kassar and Lasserre (2004)) whose timing must be chosen.

The Model

The dynamics of the temperature process, C_t, and the dynamics of the GDP process, V_t, are modelled according to the following equations:

$$dC_t = \begin{cases} a dt + \beta dW_t & \text{for} \quad t < T_L + \Delta T(k, L) \\ a(k) dt + \beta dW_t & \text{else} \end{cases} \tag{1}$$

and

$$\frac{dV_t}{V_t} = \mu dt + \sigma dB_t \tag{2}$$

Here $\{W_t, t \geq 0\}$ and $\{B_t, t \geq 0\}$ are two independent Brownian motions. The drift parameters, a and μ, and the volatility parameters, β and σ, are constant and positive. In particular, $a > 0$, the drift of the temperature process, explicitly models the Global Warming effect we are experiencing today. We also assume that the discount rate r is constant and positive.

The Model

- Climate change causes a flow of day-to-day adaptation costs over time; these costs can be viewed as levies from GDP as time goes by.
- We introduce the disposable GDP, $DGDP_t$, as the GDP, V_t, net of the day-to-day costs of climate change as moderated by adaptation efforts:

$$DGDP_t = V_t e^{-\rho(C_t - C_P)} \tag{3}$$

where $C_P = 14^\circ C$ describes the global average temperature level prior to industrialization, in absence of man-made pollution, and where $\rho > 0$ is a parameter reflecting the impact of the temperature gap and its measurement units.

The Model

- Adaptation efforts have no effect on climate dynamics; they only affect the way current temperature translates into current disposable GDP.
- Mitigation aims to slow down temperature dynamics, as represented by (1).
- An entity, such as an international organization, chooses to devote a proportion k of world disposable GDP on a yearly basis, as of T_L.
- The decision to slow down the process driving climate change is an irreversible decision that is implemented at time T_{L^*}, when the global temperature reaches some optimal threshold level L^*.

The Model

- If a fraction k is spent on mitigation as of T_L, then the temperature process will be *modified* after a given delay $\Delta T(k, L)$, i.e., at time $T_L + \Delta T(k, L)$, the trend of the temperature process will be set to $a(k)$, instead of remaining at a.
- The new temperature drift $a(k)$ is defined as:

$$a(k) = a - (a - \eta)\frac{k}{\alpha} \tag{4}$$

- In particular:

$$\lim_{k \to \alpha} a(k) = \eta \tag{5}$$

  Here η is a negative constant reflecting the self-regenerating capacity of the atmosphere.

The Model

- The delay is defined as:

$$\Delta T(k, L) = \frac{\theta}{V_{T_L} e^{-\rho(L - C_P)} k} \tag{6}$$

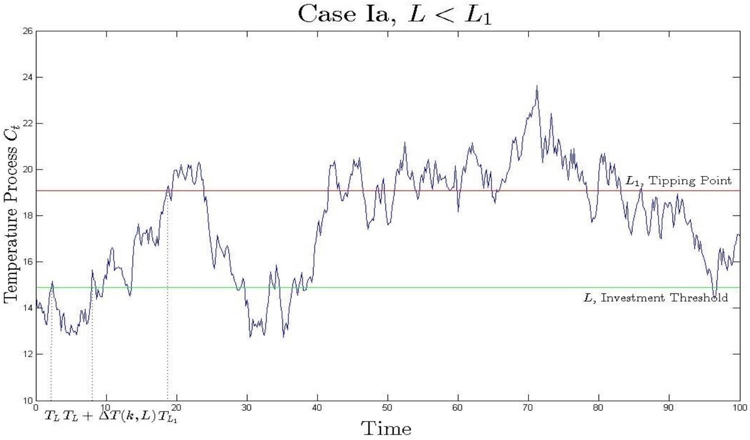
- Equation 6 models a situation in which the higher the starting disposable GDP available for expenditures in mitigation and the higher the fraction of disposable GDP actually spent, k, the quicker the effect on the temperature process.
- After $T_L + \Delta T(k, L)$, the dynamics of the temperature process C_t and of the disposable GDP process are defined by, respectively:

$$d\widehat{C}_t = a(k)dt + \beta dW_t \tag{7}$$

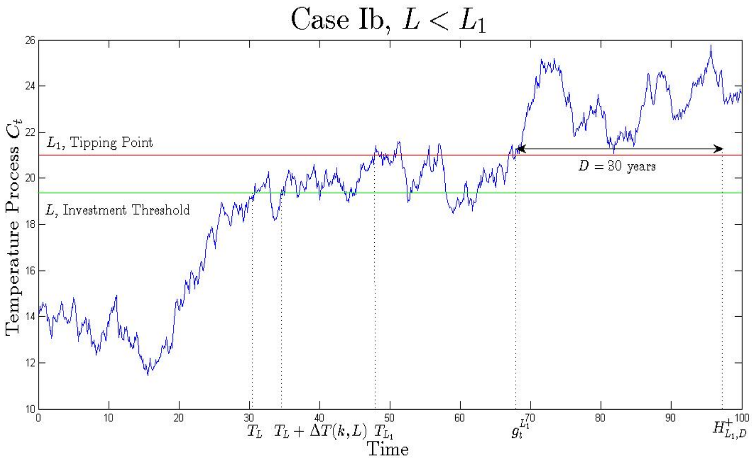
$$\widehat{DGDP}_t = V_t e^{-\rho(\hat{C}_t - C_P)} \tag{8}$$

- A global environmental catastrophe will occur if the temperature remains constantly above a given level L_1 during a period of D units of time.

The Model

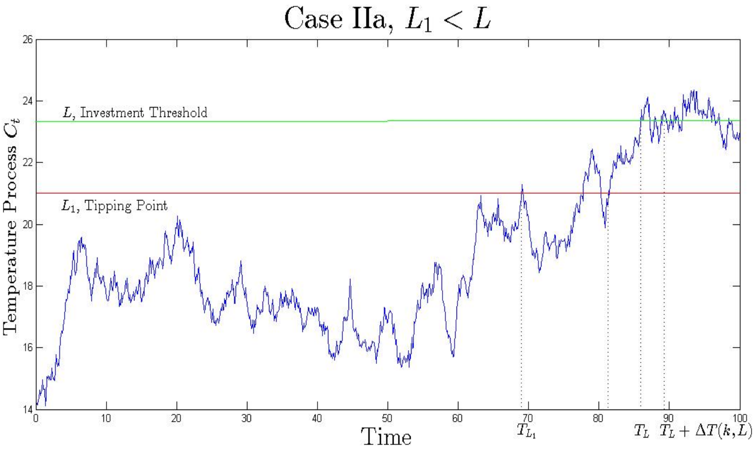

Case Ia, $L < L_1$
Temperature Process C_t
Time
26
24
22
20
18
16
14
12
10
0
10
20
30
40
50
60
70
80
90
100
L_1, Tipping Point
L, Investment Threshold
T_L $T_L + \Delta T(k, L)$ T_{L_1}

The Model



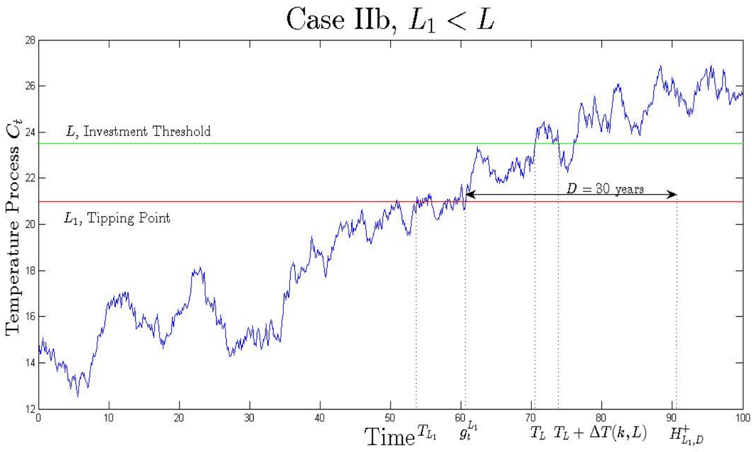
Case Ib, $L < L_1$
L_1, Tipping Point
L, Investment Threshold
$D = 30$ years
Temperature Process C_t
26
24
22
20
18
16
14
12
10
0
10
20
30
40
50
60
70
80
90
100
T_L
$T_L + \Delta T(k, L)$
T_{L_1}
$g_t^{L_1}$
$H^+_{L_1,D}$
Time

The Model



Case IIa, $L_1 < L$
L, Investment Threshold
L_1, Tipping Point
Temperature Process C_t
Time
26
24
22
20
18
16
14
0
10
20
30
40
50
60
70
80
90
100
T_{L_1}
T_L
$T_L + \Delta T(k, L)$

The Model



Case IIb, $L_1 < L$
L, Investment Threshold
L_1, Tipping Point
$D = 30$ years
Temperature Process C_t
Time
T_{L_1}
$g_t^{L_1}$
T_L
$T_L + \Delta T(k, L)$
$H^+_{L_1,D}$
28
26
24
22
20
18
16
14
12
0
10
20
30
40
50
60
70
80
90
100

The Objective

In this setting, the supremum of the expected discounted sum of future disposable GDPs corresponds to:

$$\sup_{k,L} f(k,L) \Leftrightarrow \sup_{k,L} \left[\mathbf{1}_{L<L_1} \cdot g_1(k,L) + \mathbf{1}_{L\geq L_1} \cdot g_2(k,L) \right] \tag{9}$$

with:

$$\begin{aligned} g_1(k,L) = E_{\mathbb{P}} \Bigg[& \underbrace{\int_0^{T_L \wedge T} DGDP_u e^{-ru} du}_{I_1 a} + \\ & \underbrace{\int_{T_L \wedge T}^{T_L + \Delta T(k,L) \wedge H^+_{L_1,D} \wedge T} DGDP_u \left(1 - ke^{-\delta u}\right) e^{-ru} du}_{I_1 b} + \\ & \underbrace{\int_{T_L + \Delta T(k,L) \wedge H^+_{L_1,D} \wedge T}^{H^+_{L_1,D} \wedge T} \widehat{DGDP}_u \left(1 - ke^{-\delta u}\right) e^{-ru} du}_{I_1 c} \Bigg] \end{aligned} \tag{10}$$

and

$$\begin{aligned} g_2(k,L) = E_{\mathbb{P}} \Bigg[& \underbrace{\int_0^{T_L \wedge H^+_{L_1,D} \wedge T} DGDP_u e^{-ru} du}_{I_2 a} + \\ & \underbrace{\int_{T_L \wedge H^+_{L_1,D} \wedge T}^{T_L + \Delta T(k,L) \wedge H^+_{L_1,D} \wedge T} DGDP_u \left(1 - ke^{-\delta u}\right) e^{-ru} du}_{I_2 b} + \\ & \underbrace{\int_{T_L + \Delta T(k,L) \wedge H^+_{L_1,D} \wedge T}^{H^+_{L_1,D} \wedge T} \widehat{DGDP}_u \left(1 - ke^{-\delta u}\right) e^{-ru} du}_{I_2 c} \Bigg] \end{aligned} \tag{11}$$

The Objective

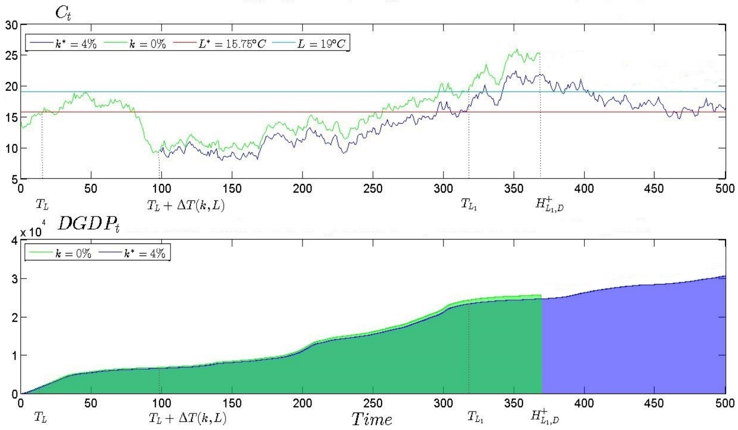


Figure 1: Impacts of Mitigation vs. Adaptation only on the Temperature Process and on the Sum of Discounted Future Disposable Gross Domestic Products, for $\beta = 0.75^\circ C$

Numerical Results

Table 1: Model Parameters

Parameter	Description	Value	Sensitivity Analysis
C_P	Pre-industrial Global Average Temperature Level	$14.0^{\circ}C$	-
C_0	2011 Global Average Temperature Level	$14.8^{\circ}C$	-
k^*	Optimal Mitigation Investment Fraction	-	[0% - 10%]
L^*	Optimal Investment Threshold	-	[14 - 22]
L_1	Catastrophe Threshold	19	-
μ	Initial Drift of the GDP Process	3.0%	-
σ	Volatility of the GDP Process	10%	-
a	Drift of the Temperature Process	$0.035^{\circ}C$	-
η	Natural Trend of Global Average Temperature	-0.1%	-
α	Parameter Modelling the Impact of Mitigation Efforts on the Temperature Drift	1	-
β	Volatility of the Temperature Process	-	$[0^{\circ}C$ - $2^{\circ}C]$
r	Discount Rate	1.5%	[0.0% - 5.0%]
δ	Depreciation Rate of the Mitigation Effort	0	-
ρ	Impact of the Temperature Gap on the Disposable GDP	0.29%	-
θ	Parameter Modelling the Magnitude of the Delay	1	-
Δt	Timesteps of the Processes	1 (year)	-
D	*Parisian* Window	50 (years)	[0; 50]
V_0	GDP in 2011	$69.993 (2011 Trillions)	-
$DGDP_0$	Disposable GDP in 2011	$69.832 (2011 Trillions)	-
T	Horizon	500 years	-

Numerical Results

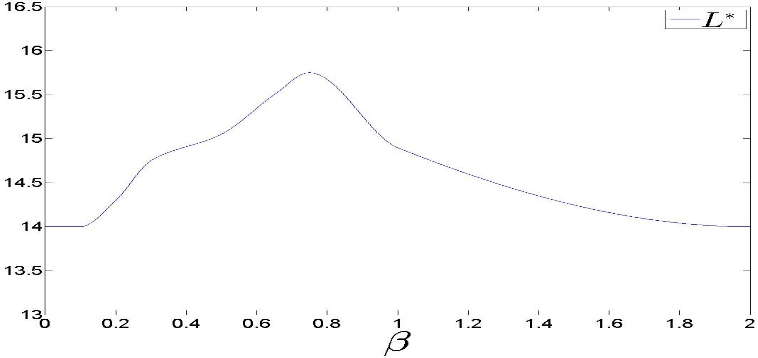


Figure 2: Optimal Investment Threshold L^*, plotted against Volatility β, for $D = 50$

Numerical Results

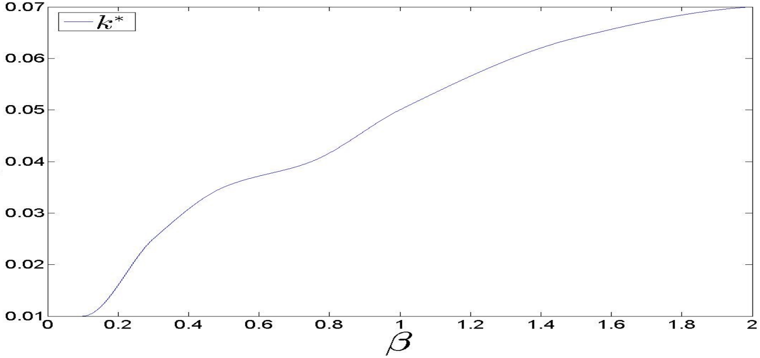


Figure 3: Optimal Mitigation Investment Fraction k^*, plotted against Volatility β, for $D = 50$

Numerical Results

- When β goes from $0°C$ to $0.75°C$, the optimal temperature threshold L increases...
- In line with the intuition that when risk increases the investment should be delayed...
- For $\beta > 0.75°C$, the optimal threshold starts decreasing again!
- In the presence of a catastrophe, higher uncertainty does not always delay the investment decision.
- The optimal fraction k^*, to be invested in mitigation when the temperature reaches L^*, is an increasing function of the volatility β.

Numerical Results

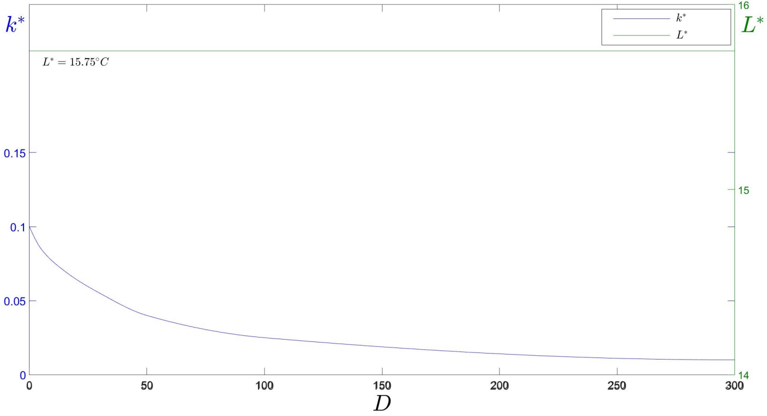


Figure 4: Optimal Mitigation Investment Fraction k^* and Optimal Investment Threshold L^*, plotted against different values of the Parisian Window D, for volatility $\beta = 0.75^\circ C$

Numerical Results

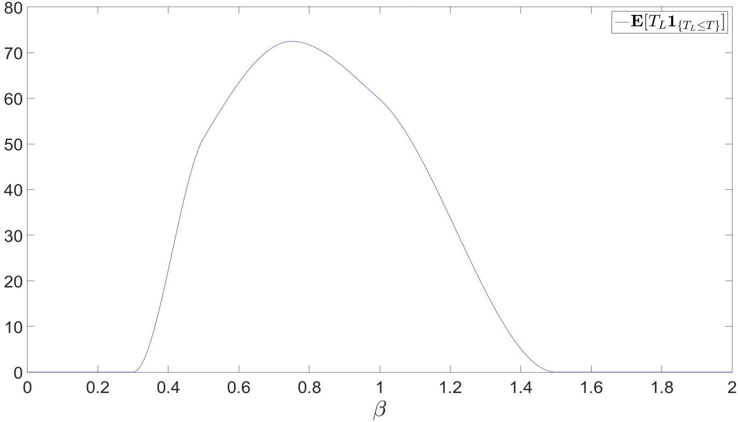


Figure 5: Expected Date T_L plotted against Volatility β, for $D = 50$

Numerical Results

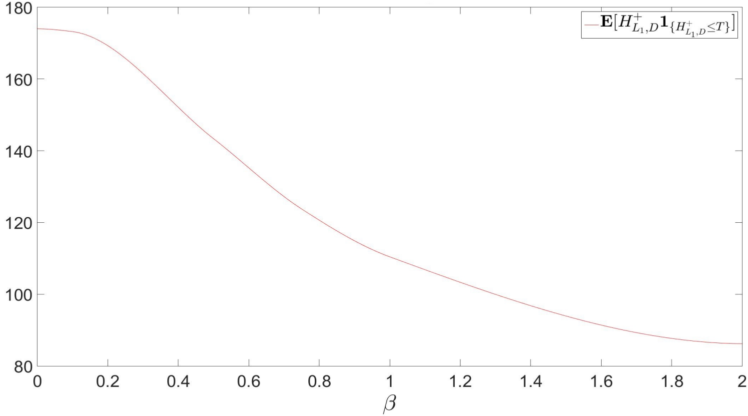


Figure 6: Expected Date of the Environmental Catastrophe $H^+_{L_1,D}$ plotted against Volatility β for $D = 50$

Numerical Results

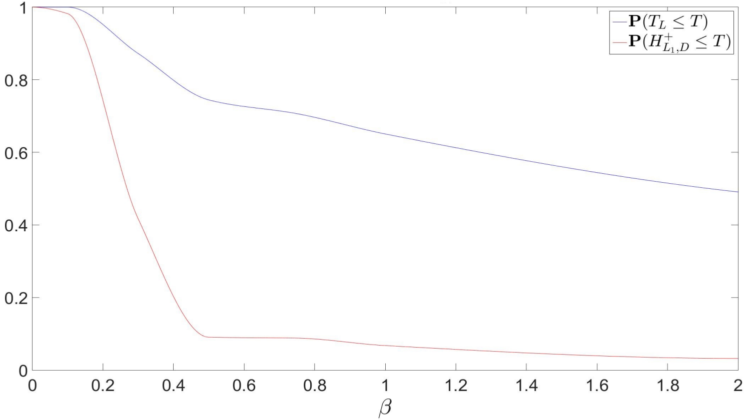


Figure 7: Probability of $T_L \leq T$ and Probability of $H^+_{L_1,D} \leq T$, plotted against Volatility β, for $D = 50$

Numerical Results

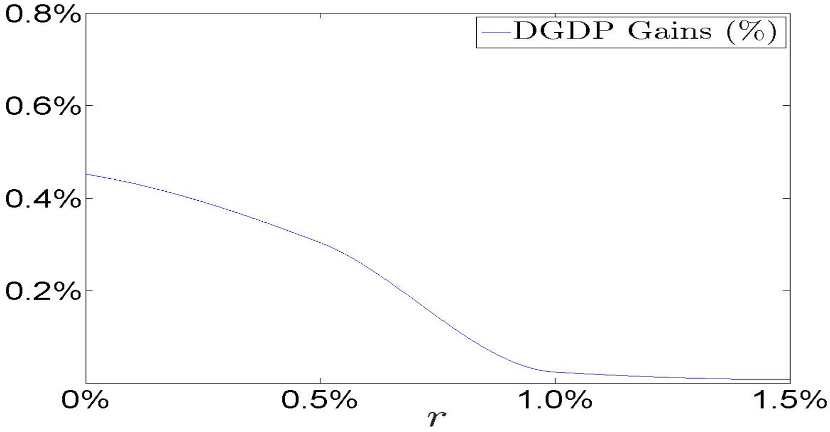


Figure 8: Cumulative DGDP Gains (%) of Mitigation vs. Adaptation only, as a function of r and for $\beta = 0.75^{\circ}C$

A Deterministic Temperature Process

When $\beta = 0$, the drift $a(k)$ becomes negative when the interest rate is small enough, i.e., when interests of future generations are taken into account. In this case, it is optimal to avoid the catastrophe. Unfortunately, only a small discount rate will generate incentives to curb CO_2 emissions and therefore decrease the drift in temperature.

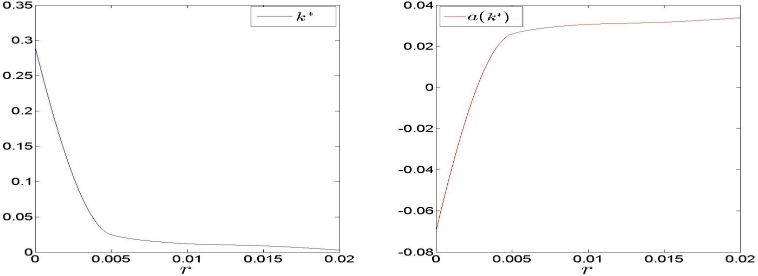


Figure 9: Optimal Mitigation Investment Fraction k^* and Temperature Drift $a(k^*)$, plotted against Interest Rate r, for volatility $\beta = 0^\circ C$ and $\alpha = 0.1$

Conclusion

- Real options models have mostly emphasized the timing of decisions.
- Our model is innovative in that the choice of the optimum date for implementing the mitigation decision is combined with the choice of the optimal mitigation effort. For a discount rate of 1.5% and with a temperature volatility of $0.3^{\cdot}C$, the model shows that governments should invest 2.5% of disposable GDPs in mitigation when the temperature process hits $14.75^{\cdot}C$ in order for the world to achieve the maximum possible expected discounted sum of future disposable GDP.
- Unfortunately, global temperature level has already reached this threshold.

Conclusion

- Mitigation is urgently needed. Adaptation is not a substitute for mitigation. The effort that needs to be extended will be higher the longer mitigation is postponed.
- The optimum proportion of GDP that should be invested in mitigation lies between 1% and 7%. This proportion is much higher than currently observed levels. It is also much higher than the exogenous adaptation estimated in this model.
- The relationship between the optimal temperature threshold and temperature volatility is not monotonic; the optimal threshold diminishes when the assumed volatility increases beyond a given level.

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3) Climate change and real options: the micro level

Climate change investment strategies in the presence of uncertainty

Marc Chesney, working paper, 2020

The main features of the model

- A general comment:

In quantitative finance, as soon as a stochastic process is used, it is often assumed that its parameters are known. It might be a strong assumption, in particular in the field of global warming mitigation, in which financial processes are less documented.

- First feature:

Not all the parameters of the underlying dynamics are known by the investor.

- Second feature: A mean-reverting process is used in order to model the underlying dynamics.

With a standard geometric Brownian motion, the mean process is monotonic. A mean-reverting process is more realistic.

The context

- The example of an investment in wind energy.

- Here is the net cash flow process generated by such an investment:

$$dX_u = k(\theta - X_u)du + kdW_u$$

The potential investor installs devices in a selected field in other to measure wind speed and to assess expected future net cash flows.

Confronted with incomplete information, he does not know the mean-reverting level θ, but will try to optimally assess it over time.

- At a given time t, its estimate of this parameter is the observed average until this time:

$$\hat{\theta}_t = \frac{\int_0^t X_u du}{t}$$

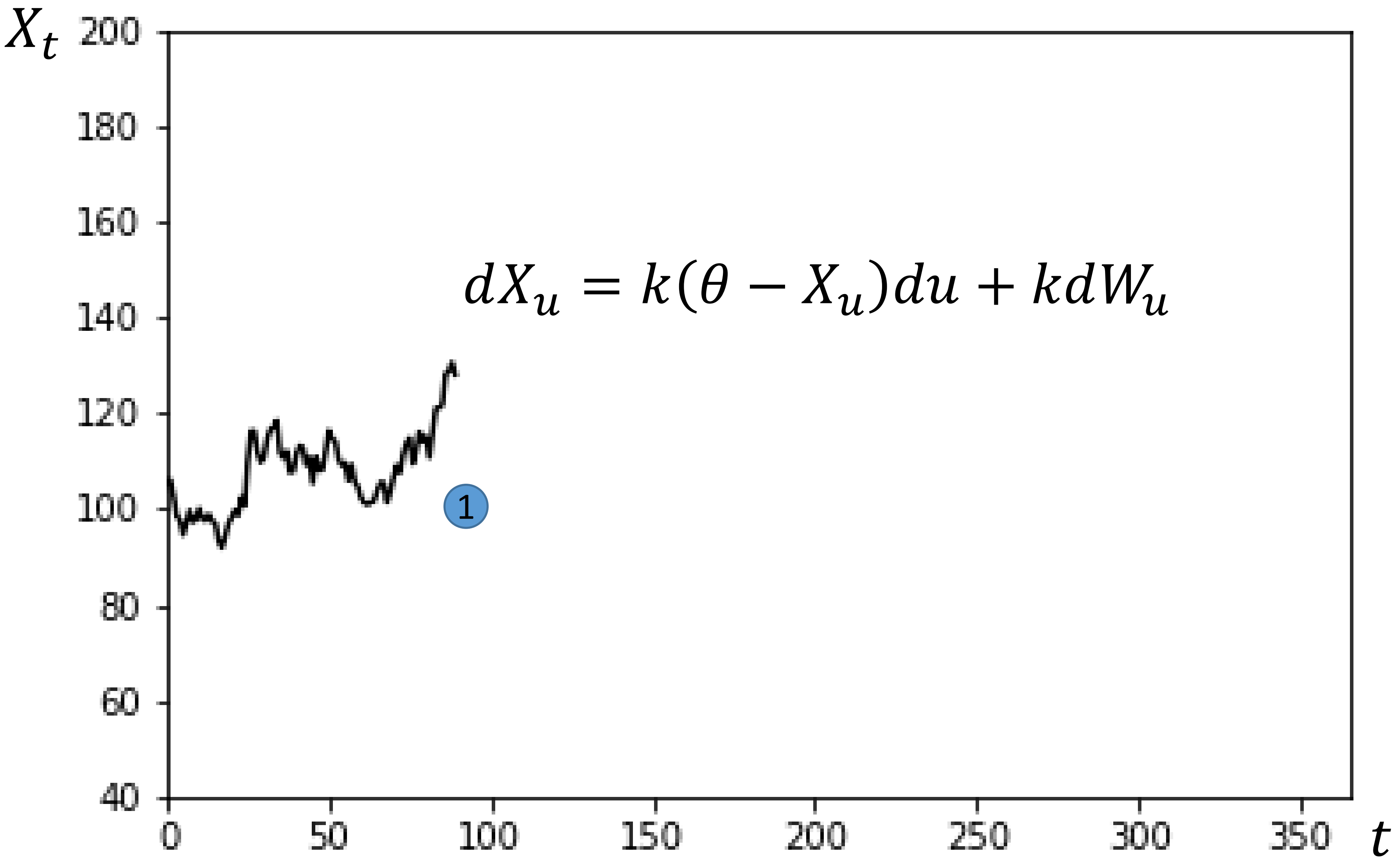
The investor is confronted with a trade-off: waiting before investing in order to better assess this unknown parameter or stop waiting in order to start receiving the corresponding net cash-flows.

- In a perpetual setting the investor maximizes the expected utility of the sum of discounted net cash-flows.

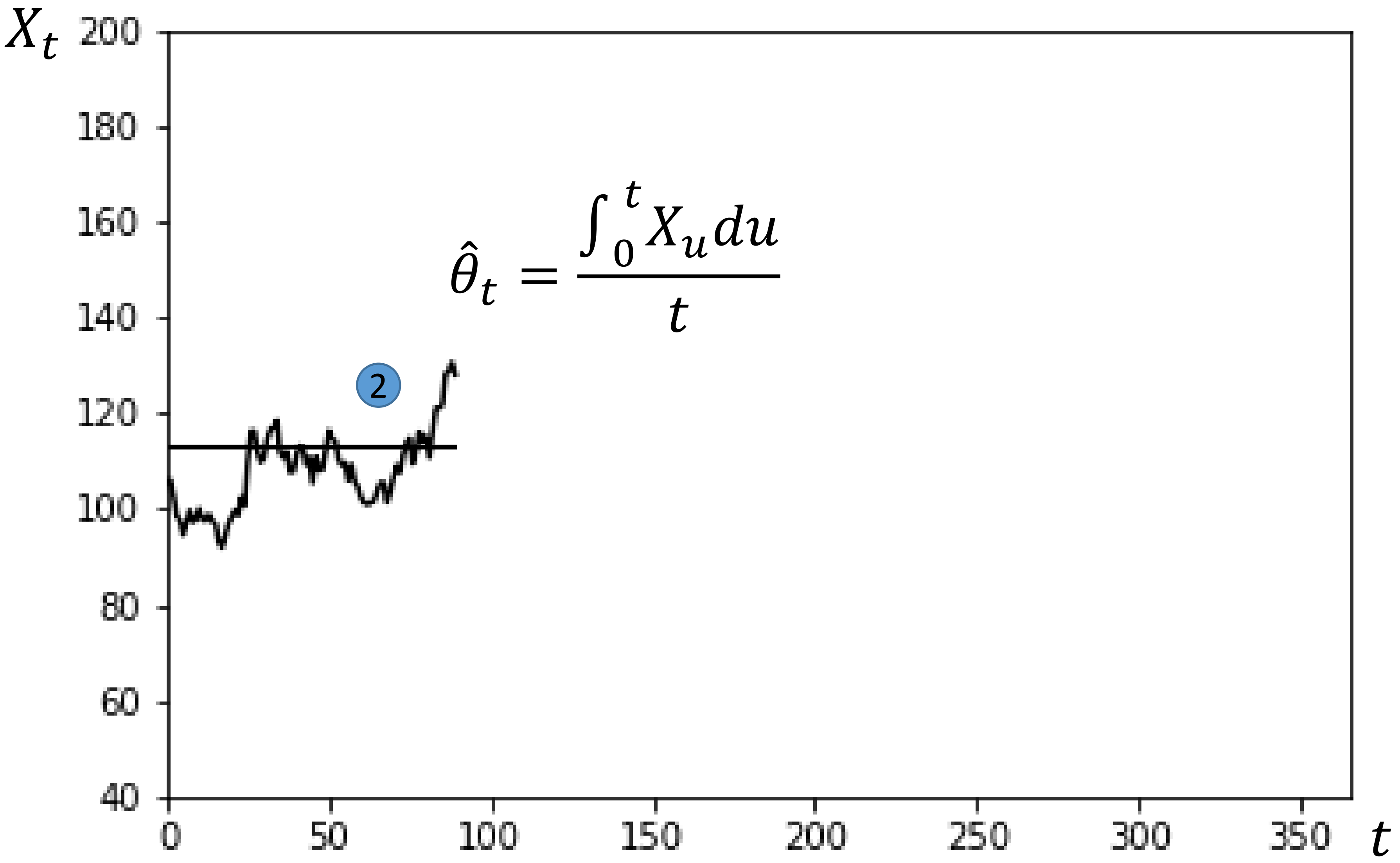
$$\mathrm{f}\left(X_t, \frac{\int_0^t X_u du}{t}, t\right) = \sup_{\tau \, \epsilon \, \Gamma} \mathbb{E}_{\mathbb{P}} \left(U\left(\int_\tau^{\tau+\Delta T} e^{-ru} \; X_u du \right) \Big| \mathcal{F}_\tau \right)$$

- This model answers the above-mentioned question: when, in a context of uncertainty. It is an investment decision under uncertainty.

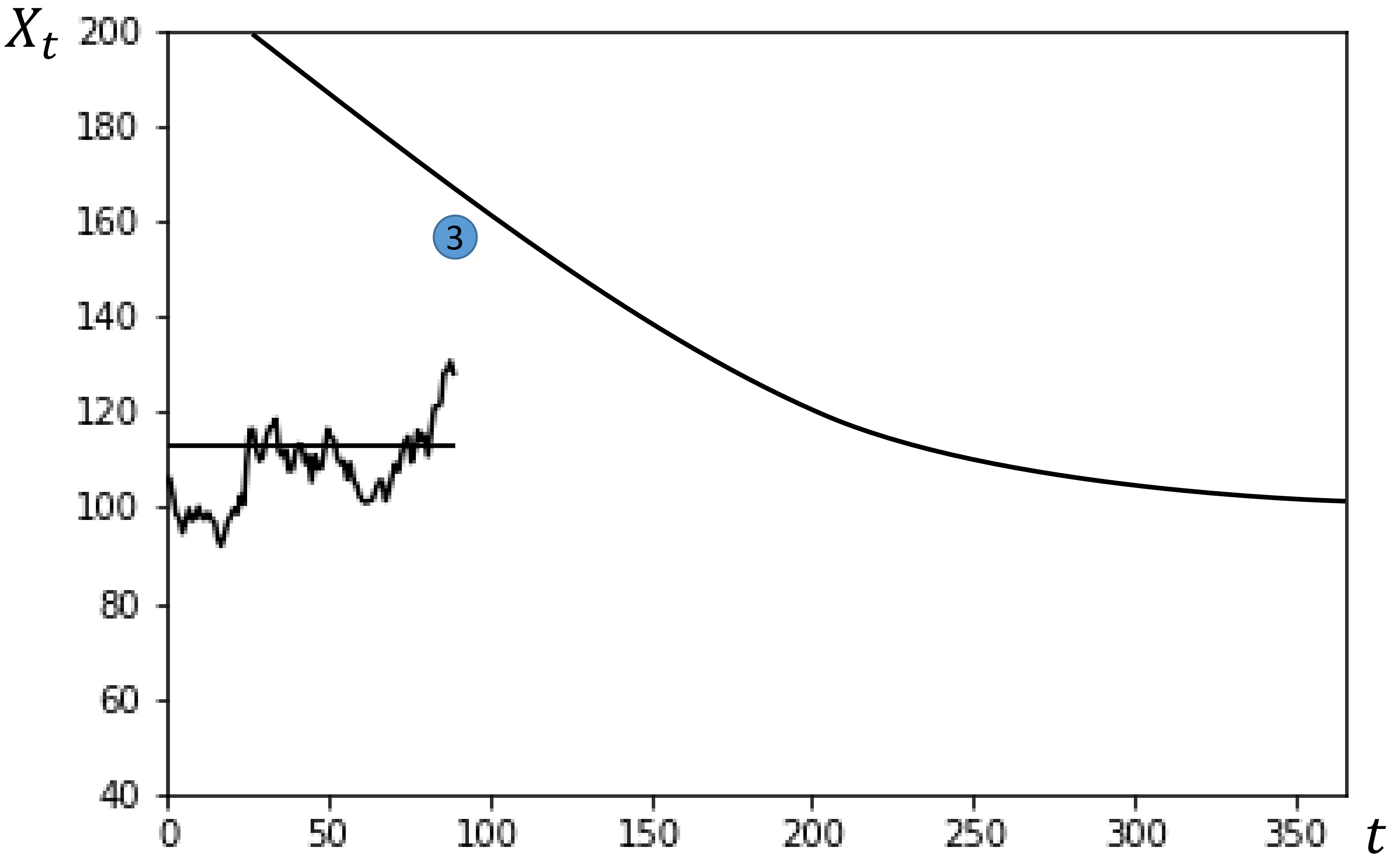
See: *Some Combinations of Asian, Parisian and Barrier Options*, M. Yor, M. Chesney, H. German, and M. Jeanblanc-Picqué, In Mathematics of Derivative Securities, *Cambridge University Press*, 1997

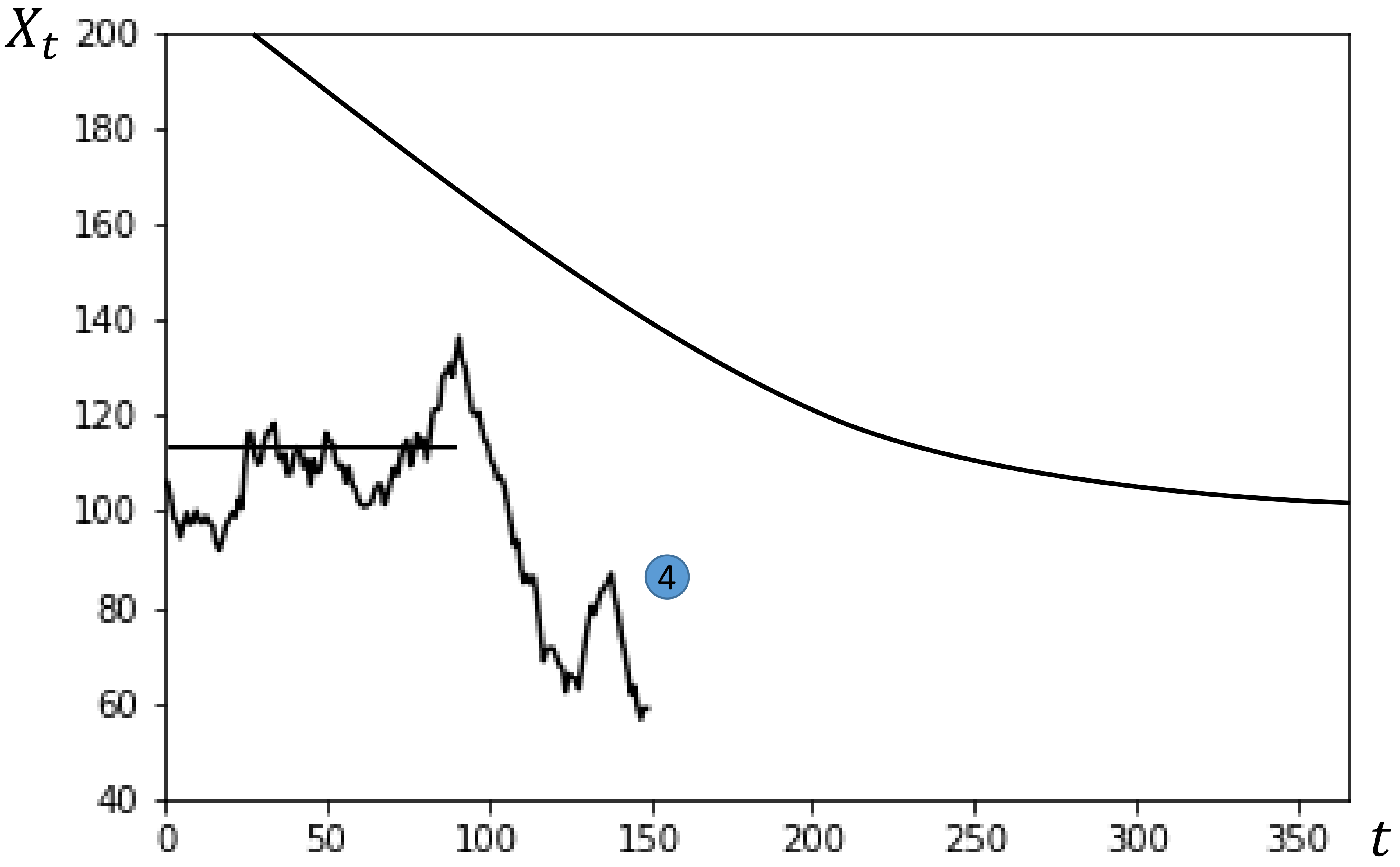


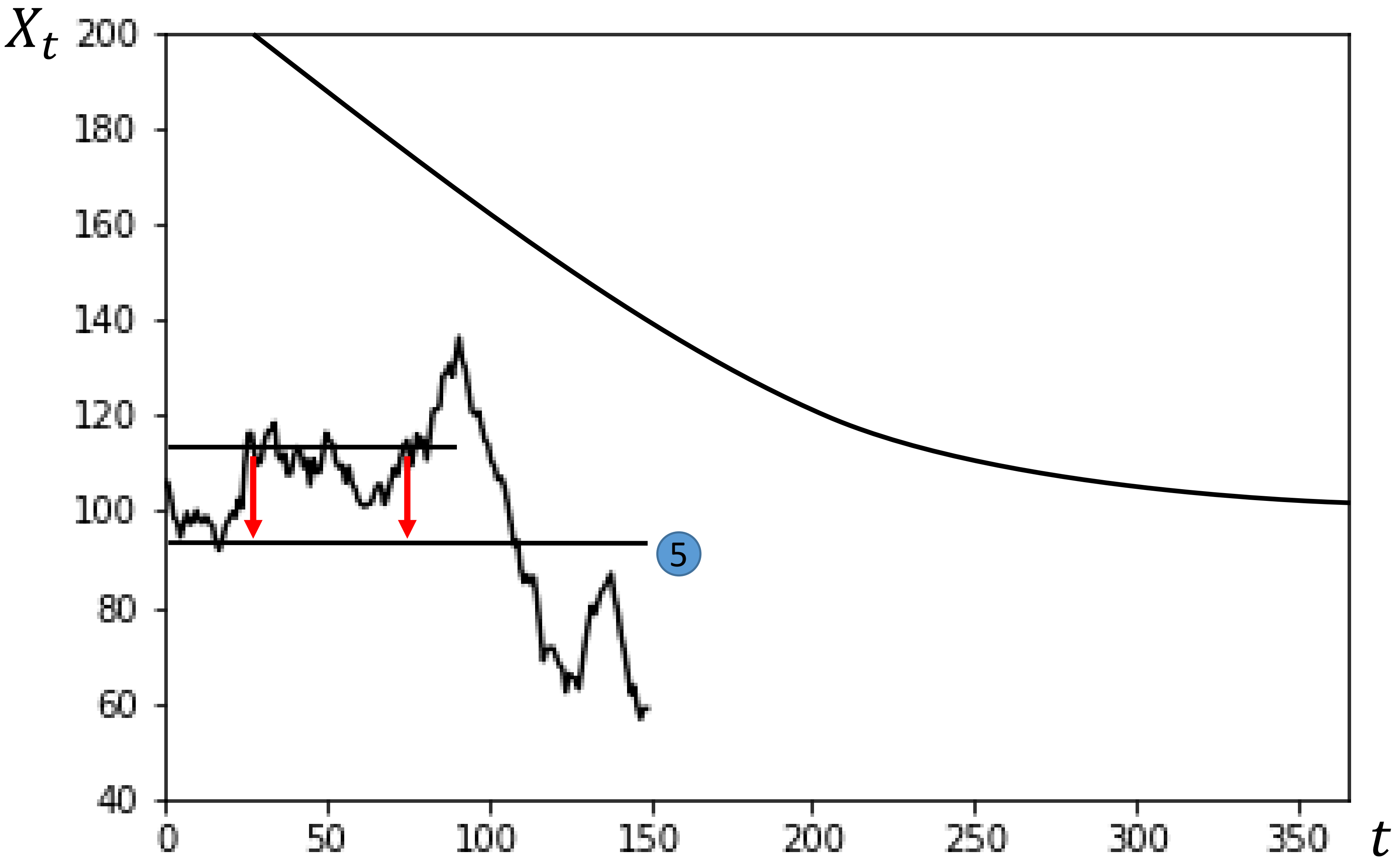
X_t
200
180
160
140
120
100
80
60
40
0
50
100
150
200
250
300
350
t
$dX_u = k(\theta - X_u)du + kdW_u$
1



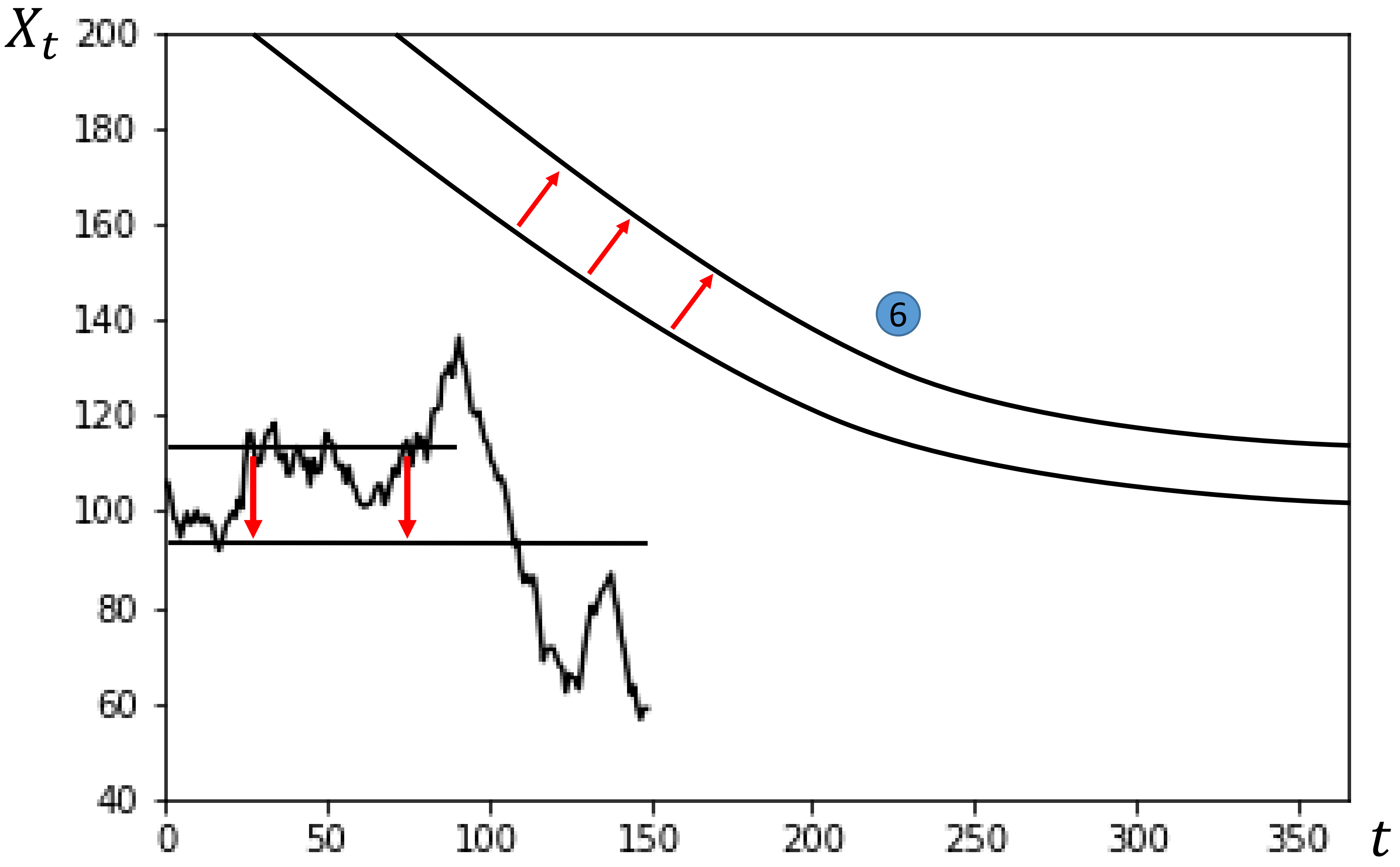
$$\hat{\theta}_t = \frac{\int_0^t X_u du}{t}$$


X_t
200
180
160
140
120
100
80
60
40
0
50
100
150
200
250
300
350
t
3


X_t
200
180
160
140
120
100
80
60
40
0
50
100
150
200
250
300
350
t
4



X_t
200
180
160
140
120
100
80
60
40
0
50
100
150
200
250
300
350
t
5



X_t
200
180
160
140
120
100
80
60
40
0
50
100
150
200
250
300
350
t
6

Thank you for your attention!